Quantum Teleportation with a 3-Bell-state Analyzer

J.A.W. van Houwelingen†, N. Brunner, A. Beveratos, H. Zbinden, and N. Gisin

Group of Applied Physics, University of Geneva, Switzerland

We present a novel Bell-state analyzer for time-bin qubits allowing the detection of three out of four Bell-states with linear optics, only two detectors and no auxiliary photons. The theoretical success rate of this scheme is 50%. A teleportation experiment was performed to demonstrate its functionality. We also present a teleportation experiment with a fidelity larger than the cloning limit.

A Bell-State Analyzer (BSA) is an essential part of quantum communications protocols such as a quantum relay based on quantum teleportation [1, 2, 3, 4], entanglement swapping [5] or quantum dense coding [6, 7]. It has been shown that, using only linear optics, a BSA for qubits has a maximal success rate of 50% when no auxiliary photons are used [8, 9]. This, however, does not limit the number of Bell-states one can measure, but only the overall success rate. A complete BSA could be achieved using either nonlinear optics or using continuous variable encoding [10, 11]. However, each of these two alternatives carry some significant drawback. The nonlinear optics approach has exceedingly low efficiency [12]; while continuous variable encoding has the disadvantage that postselection is not possible. Note that postselection is a very useful technique that allows one to use only 'good' measurement results and straightaway eliminate all others [13].

Today's optimal BSA schemes based on linear optics for qubits are only able to detect two out of four Bell-states [5, 14, 15, 16]. Here we present a novel scheme for a BSA which achieves the 50% upper bound of success rate, but can distinguish three out of the four Bell-states while using only two detectors. We demonstrate this scheme in a quantum teleportation experiment at telecom wavelengths. The new BSA is inspired by, although not limited to, the time-bin implementation of qubits and is thus fully compatible with the field of Quantum Communications [17].

At first, one may think that detecting 3 out of 4 Bell-state provides a full BSA. Indeed, if the BSA would consist of a standard Von Neumann projective measurement, then the fourth Bell-state would merely correspond to the non-detection of the 3 others. But our BSA is a new example of the power of generalized quantum measurements, it uses a Positive Operator Valued Measurement (POVM) with 21 possible outcomes. Some outcomes of this POVM (see Fig. 1) correspond to one of the 3 Bell-

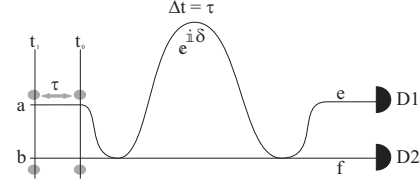†email: jeroen.vanhouwelingen@physics.unige.ch

FIG. 1: A schematic representation of the new type of Bell-state Measurement. When two qubit states are sent into a time-bin interferometer the output state is a mixture of photons in two directional modes and three temporal modes. By looking at certain combinations of these photons a Bell-state Measurement can be performed for three different Bell-States

states that can be distinguished unambiguously and thus detect this state. The others correspond to inconclusive results. More specifically, the Bell-state $|\psi_+\rangle$ is always detected, $|\phi_-\rangle$ is never detected, while $|\psi_-\rangle$ and $|\phi_+\rangle$ are detected with a 50% success rate.

In previous BSAs the main method was to use a beamsplitter followed by detectors to determine the input Bell-state. We replace this simple approach by a time-bin interferometer equivalent to the ones used to encode and decode time-bin qubits (Fig. 1) [17]. The two photons of the Bell-state enter in port a and b, respectively. In a BSA using only a beamsplitter, one is able to distinguish between $\psi_\pm$ but not $\phi_\pm$ since in the later case the two photons will experience photon bunching but this interference does not contain the phase information that distinguishes ϕ_- from ϕ_+. In our case, the first beamsplitter acts like above, but we introduce a second interference possibility between the two photons on the second beamsplitter. This second interference allows one to distinguish more than just two Bell-states.

The output coincidences in ports e and f, i.e. on detectors $D1$ and $D2$, are summarized in table I as a function of the input state. By convention, a coincidence at time '0' means that the photon did not accumulate any delay with regards to its creation pulse. This is only possible if it took the short path in the BSA and it was originally a photon in time-bin t_0 (Fig. 1). A coincidence at time '1' means that the photon was originally in t_1 and took the short path of the BSA interferometer or it was in t_0

| D1 | 00 | 11 | 22 | 01 | 02 | 12 | | | | | | | 0 | 2 | 1 | 1 | 0 | 1 | 2 | 0 | 2 |
D2							00	11	22	01	02	12	0	2	1	0	1	2	1	2	0
$\lvert\phi_+\rangle$	1/16	1/16					1/16	1/16					1/8	1/8	**1/2**						
$\lvert\phi_-\rangle$	1/16	1/16			1/4	1/4	1/16	1/16					1/8	1/8							
$\lvert\psi_+\rangle$				**1/8**	**1/8**					**1/8**	**1/8**					**1/8**	**1/8**	**1/8**	**1/8**		
$\lvert\psi_-\rangle$		1/4	1/4								**1/8**	**1/8**								**1/8**	**1/8**

TABLE I: The table shows the probability to find any of the 21 possible coincidences as a function of the input Bell-State. A '0' in row D1 means that a photon was found at detector 'D1' and at a time corresponding to the photon having taken the short path in the interferometer and it was originally a photon in time-bin t_0, a '1' corresponds to $t_0 + \mathbf{1} \times \tau$ with τ corresponding to a the difference between the time-bins etc. Note that several combinations of detection are possible for only one Bell-state(the bold entries), therefor when such a combination is found a Bell-state Measurement was performed. The theoretical probability of a successful measurement is 0.5 which is the optimal value using only linear optics [8].

and took the long path. A coincidence at time '2' then means the photon was in t_1 and took the long path.

In table I one can distinguish two cases. Either the result is unambiguous in which case we have successfully distinguished a Bell-state. Or the result could have been caused by two specific Bell-states, i.e. the result is ambiguous and hence inconclusive.

The above described approach is correct in the case were the phase δ of the BSA interferometer is set to 0. Experimentally, it is not trivial to align this phase because the pump-interferometer and the BSA interferometer involve different wavelengths. For an arbitrary phase δ the situation is slightly different and interesting. In such a case, our BSA still distinguishes 3 Bell-states, but these are no longer the Bell-states of the computational basis. For a teleportation experiment this means the basis for the measured Bell-states is not the same as the basis for the entangled states shared between Bob and Charlie. Still, perfect teleportation is possible, but with the difference that the unitary transformations that Bob has to apply after receiving the classical information about the result of the BSA have changed and no longer include the identity: all unitary transformations are non-trivial but they remain experimentally feasible. More specifically, the analyzed Bell-states are:

$$\phi'_\pm = \lvert 00\rangle \pm e^{2i\delta}\lvert 11\rangle \qquad (1)$$
$$\psi'_\pm = e^{i\delta}(\lvert 01\rangle \pm \lvert 10\rangle) \qquad (2)$$

These new Bell-states are equivalent to the standard states except that the $\lvert 1\rangle$ is replaced by $e^{i\delta}\lvert 1\rangle$ for each of the input modes. Therefore the unitary transformations that have to be applied to retrieve the original state of the teleported photon also have to be modified from $[I,\ \sigma_z,\ \sigma_x,\ \sigma_z\sigma_x]$ to $[\sigma_{2\delta},\ \sigma_z\sigma_{2\delta},\ \sigma_x,\ \sigma_z\sigma_x]$. Here $\sigma_{2\delta} = e^{-2i\delta}P_{\lvert 1\rangle} + P_{\lvert 0\rangle}$ is a phase shift of 2δ to be applied to the time bin $\lvert 1\rangle$.

In a more realistic experimental environment the success probabilities of the BSA will be affected by detector limitations, because existing photon detectors are not fast enough to distinguish photons which follow each other closely (in our case two photons separated by

$\tau = 1.2ns$) in a single measurement cycle. This limitation rises from the dead time of the photodetectors. When including this limitation we find that the maximal attainable probabilities of success in our experimental setup are reduced to 1/2, 1/4 and 1/2 for ψ_+, ψ_- and ϕ_+ respectively. This leads to an overall probability of success of 5/16 which is greater by 25% than the success rate of 1/4 with a BSA consisting only of one beamsplitter and two identical detectors.

In order to demonstrate successful Bell-state analysis we performed a teleportation experiment. A schematic of the experimental setup is shown in Fig. 2. Alice prepares a photon in the state $\lvert \zeta_A\rangle = \lvert 0\rangle + e^{i\alpha}\lvert 1\rangle$. Bob analyzes the teleported photon and measures interference fringes for each successful BSA announced by Charlie. The setup consist of a mode-locked Ti-sapphire laser creating 150 fs pulses with a spectral width of 4nm, a central wavelength of 711nm and a mean power of 400mW. This beam is split in two beams using a variable coupler ($\lambda/2$ and a PBS). The reflected light (Alice) is sent to a Lithium tri-Borate crystal(LBO, Crystal Laser) were by parametric down-conversion a pair of photons is created at 1.31 and 1.55 μm. Pump light is suppressed with a Si filter, and the created photons are collected by a single mode optical fiber and separated with a wavelength-division-multiplexer (WDM). The 1.55 μm photon is ignored whereas the 1.31 μm is send to a fiber interferometer which encodes the qubit on the equator of the Bloch sphere. In the same way, the transmitted beam (Bob) is send onto another LBO crystal after having passed through an unbalanced Michelson bulk optics interferometer. The non-degenerate entangled photons produced in this way corresponds to the ϕ_+ state. The photons at 1.31 μm are send to Charlie in order to perform the Bell-state measurement. In order to assure temporal indistinguishability, Charlie filters the received photons down to a spectral width of 5 nm. In this way the coherence time of the generated photons is greater than that of the photons in the pump beam, and as such we can consider the photons to be emitted at the same time. Bob filters his 1.55 μm photon to 15 nm in order to avoid multi-photon events [18, 19]. A liquid Nitrogen cooled Ge Avalanche-
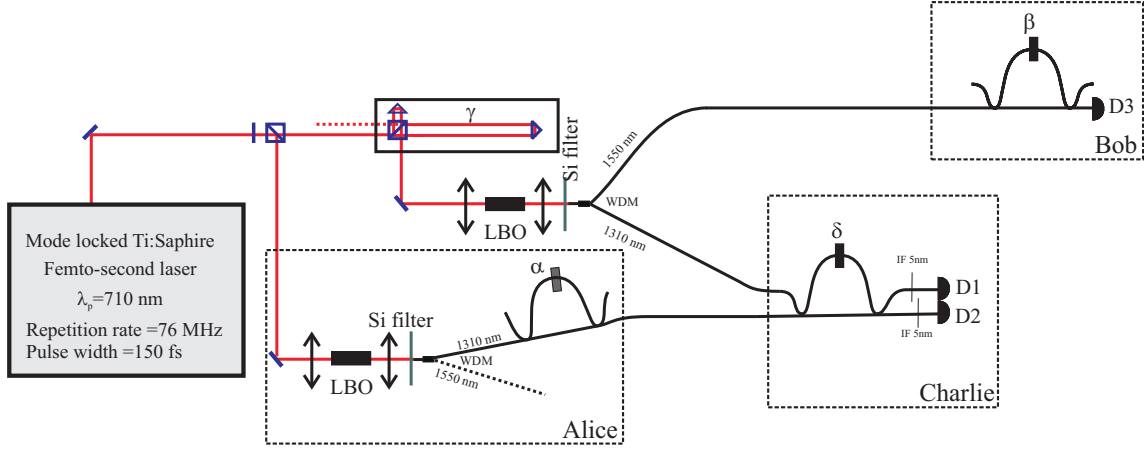
FIG. 2: A rough overview of the experimental setup. The fiber interferometers shown here are in reality michelson-interferometers, for the interferometer in the BSA two circulators are used to have two separate inputs and outputs. Not shown in the figure is the method used for stabilizing the interferometers.

Photon-Detector (APD) D1 with passive quenching detects one of the two photons in the BSA and triggers the InGaAs APDs (id Quantique) D2 and D3. Events are analyzed with a time to digital converter (TDC, Acam) and coincidences are recorded on a computer.

Each interferometer is stabilized in temperature and an active feedback system adjusts the phase every 100 seconds using separate reference lasers. In this way the quantum teleportation scheme works with independent units and is ready for 'in the field' experiments. A more detailed description of the active stabilization is given in ref [5].

The temporal indistinguishability of the photons arriving at the BSA is usually tested by measuring a Hong-Ou-Mandel dip. In our BSA this is not directly possible. Photons that have bunched on the first interferometer will be split up by the second interferometer in a nondeterministic manner and as such there will be no decrease in the number of coincidences when looking at the same time of arrival. There will actually be an increase for these coincidences because the amount of photons that took a different path in the interferometer will decrease. In our experimental setup this means one has to look at an increase in coincidence rate at times '00' or '22'. A typical result from this alignment procedure can be found in (Fig. 3). The expected antidips have a net visibility of 32% and 26% when subtracting noise. The maximal attainable value is 1/3 due to undesired but unavoidable double-pair events [20].

First we performed a quantum teleportation with independent units using a beam splitter for the Bell-state measurement. This enables us to test our setup in terms of fidelity. In order to check the fidelity of the teleported sate, Bob sends the teleported photon through an analyzing interferometer and measures the interference fringes conditioned on a successful BSA. When Bob scans the
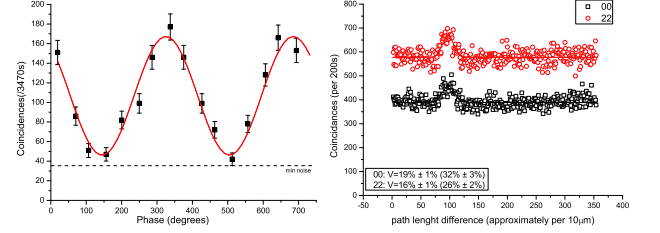

FIG. 3: *Left:* The Result of the 1-Bell state teleportation experiment (a beamsplitter instead of the interferometer) with $F_{raw} = 79\%$ and $F_{net} = 91\%$. *Right:* A typical result of the scan in delay of the coincidences SS and LL. The predicted antidip is clearly visible in both curves with a visibility after noise substraction of 32% for '00' and 26% for '22' (max=1/3).

phase of his interferometer we obtained a raw visibility of $V = 57\%$ ($F = 79\%$) and a net visibility of $V = 83\% \pm 4$ ($F = 91\% \pm 2$) clearly higher than the cloning limit of F= 5/6 (Fig. 3). We then switched to the new BSA. This new setup introduces about 3dB of excess loss, due to added optical elements including the interferometer and its stabilization optics. These losses result in a lower count rate. For experimental reasons we now scan the interferometer of Alice instead of Bob. The experiments were performed for approximately 4.4 hours per point in order to accumulate enough data to have low statistical noise. The expected interference fringes after Bob's interferometer are of the form $1 \pm \cos(\alpha + \beta - \gamma)$ for a projection on $\psi_{\pm}$ and $1 + \cos(\alpha - \beta + \gamma - 2*\delta)$ for a projection on ϕ_{+}. Hence one would expect to find three distinct curves, two with a phase difference of π, and the third dephased by $-2*(\beta - \gamma + \delta)$. Where α, β, γ and δ are the phases of Alice, Bob, the entangled pair preparation interferometer and the BSA measurement interferometer respectively. Note that one can set the phase γ to 0 as reference phase and that Bob is able to derive the phase value δ of the BSA interferometer just by looking at the
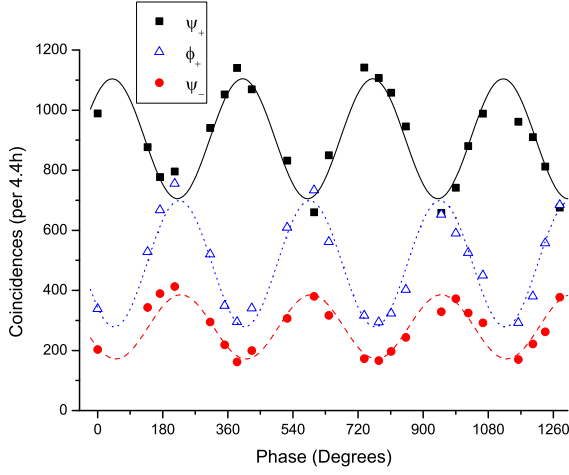
FIG. 4: These graphs show the interference fringes found when scanning the interferometer at Bob. we found visibilities of 0.22,0.43 and 0.38 for $|\psi_+\rangle$, $|\phi_+\rangle$ and $|\psi_-\rangle$. The average visibility of the BSA is $V_{avg} = 0.34$ (F=0.67). If we subtract the noise (not shown) we find visibilities of 0.32,0.50 and 0.71 for $|\psi_+\rangle$, $|\phi_+\rangle$ and $|\psi_-\rangle$ which leads to an average of $V_{avg} = 0.51$ ($F = 0.76$)

phase differences between the fringes made by $\psi_\pm$ and ϕ_+ and his knowledge about β.

In Fig. 4 we show the coincidence interference fringes between Bob and a successful BSA. As expected fringes for $|\psi_-\rangle$ and $|\psi_+\rangle$ have a π phase difference due to the phase flip caused by the teleportation. On the other hand fringes for $|\psi_+\rangle$ and $|\phi_+\rangle$ are dephased by $-2(\beta+\delta)$ which in this case we had arranged to be approximately 0. The raw visibilities obtained for the projection on each Bell-state are $V_{\psi_-} = 0.38$, $V_{\psi_+} = 0.22$, $V_{\phi_+} = 0.43$ which leads to an overall value of $V = 0.34$ ($F = 0.67$). In order to check the dependence of $|\phi_+\rangle$ on δ we also performed a teleportation with a different value and we clearly observe the expected shift in the fringe (Fig. 5) while measuring similar visibilities.

Finally the authors would like to stress two points about this novel BSA. First it is possible to implement this BSA for polarization encoded photons by creating a second interference possibility for H- and V-polarization. This would require 4 detectors, but an overall efficiency of 50% can be achieved with current day detectors. The limit of 50% can also be achieved with time-bin encoded qubits but would require the use of ultra fast optical switches and two more detectors. We did not implement this due to the losses associated with introducing current day high speed integrated modulators. Secondly, even though three out of four Bell-states can be distinguished, one can not use this scheme in order to increase the limit of $\log_2 3$ bits per symbol for quantum dense coding.

In conclusion we have shown experimentally that it is possible to perform a three-state Bell analysis while using only linear optics and without any actively controlled local operations on a single qubit. In principle
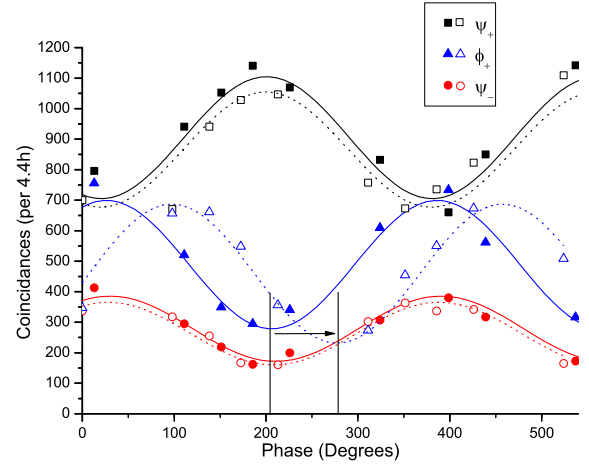

FIG. 5: This graph shows two measurements that were taken with a different value of δ. There is a clear shift of the fringe for $|\phi_+\rangle$ with regards to the other fringes.

this measurement can obtain a success rate of 50%. We have used this BSA to perform a teleportation experiment, and obtained a non-corrected overall fidelity of 67%, after noise substraction we find F=76%. Also, we performed a teleportation experiment with a one state BSA which exceeded the cloning limit.

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